

## Lecture 15

In this lecture we finish our introductory look at logic, and we begin our study of algorithms.

# Equivalence



We say  $P$  is equivalent to  $Q$  and

write

$$P \Leftrightarrow Q$$

if whenever  $P$  is true,  $Q$  is true, and also  
whenever  $Q$  is true,  $P$  is true.

Note: some texts use



instead of  $\Leftrightarrow$ .

In words:

$P$  is equivalent to  $Q$

$P$  if and only if  $Q$

iff

$P$  is necessary & sufficient for  $Q$

---

Eg.

Which of the following are true?

- Some integers are real numbers. True
- "Most" integers are real numbers. True?
- All integers are real numbers. True
- Some real numbers are irrational. True
- All real numbers are irrational. False

Note

$P \iff Q$  means

$$\left\{ \begin{array}{l} P \Rightarrow Q \\ \text{and} \\ P \Leftarrow Q \end{array} \right.$$

|  
"is implied by"

Eg.

$$3 > 2$$

$$\iff$$

$$2 < 3$$

[True]

$$2 < 3$$

$$\iff$$

$$2 < 1$$

[False]

# Algorithms

As previously mentioned, an algorithm is a systematic procedure for solving some kind of problem.

Algorithms are basic to computer programs. Essentially, a program implements one or more algorithms.

This is why algorithmic complexity is important. (But it's a difficult topic.)

We will study a few algorithms.

## The Division Algorithm

Let  $a, b$  be integers. Suppose  $b > 0$ .

Then there exist integers  $q$  and  $r$  such that

$$a = bq + r$$

where  $0 \leq r < b$ .

We call  $q$  the quotient and  $r$  the remainder in the division of  $a$  by  $b$ .

The process of expressing  $a$  in this way is the application of the division algorithm.

Essentially this says that we can divide one integer by another if the latter is positive, and that we get a quotient and a remainder.

If  $a > 0$ , then

$$q = \lfloor a/b \rfloor. \quad (\text{floor of } a/b).$$

E.g.  $a = 31, b = 7$

$$q = \lfloor 31/7 \rfloor = \lfloor 4\frac{3}{7} \rfloor = \lfloor 4.\dot{4}28571 \rfloor = 4$$

So  $a = bq + r$

gives  $31 = 7 \cdot 4 + \boxed{3}$ .

Valid input requires  
 $a, b$  to be integers  
and  $b > 0$ .

Given  $a, b$ :

$$q = \lfloor a/b \rfloor$$

$$r = a - bq$$

---

### Euclidean Algorithm

Let  $a, b$  and  $c$  be integers.

Suppose that  $ab = c$ .

Then  $c$  is the product of  $a$  and  $b$ .

Also,  $a$  and  $b$  are divisors or factors of  $c$ .



We also say that  $c$  is a multiple of  $a$  (and also of  $b$ ).

Let  $m, n$  be positive integers.

A positive integer  $c$  is a common factor or common divisor of  $m$  and  $n$  if it divides (is a divisor, or factor, of) both of them.

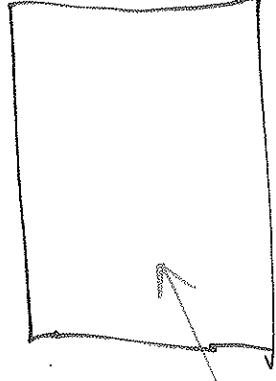
A positive integer  $d$  is a common multiple of  $m$  and  $n$  if it is a multiple of both of them.

The gcd (greatest common divisor) of  $m$  and  $n$  is the greatest no. which is a common divisor of both of them.

It's also called the highest common factor or hcf.

E.g. 18, 24

$$\gcd(18, 24) =$$



Answer is 6.

There is a systematic procedure for getting the gcd. It's the Euclidean algorithm.